

Topics Include:

Circles and Spheres • Congruent Triangles and Transformations

Exploring Geometry: Points, Lines, and Angles in a Plane

Parallel Lines and Transversals • and more



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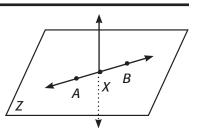
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Points, Segments, Rays, Lines, and Planes

In geometry, figures are created using points, lines, line segments, rays, and planes. Each item has a unique and specific definition, each a certain way to express it using symbols, and each a certain way the symbols are translated into words.



The figure to the right contains points, segments, lines and planes. Use the figure to complete the chart below.

Type of Figure	Symbol	Words	Drawing
Point	Point A		
Line		Line AB	← ← ← → A B
	ĀB		A B
Ray		Ray AB	
	□Z	Plane Z	

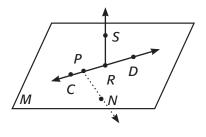
Each figure also has a specific definition. Identify each type of figure. Complete each definition using the chart and figures above.

Rules for Naming Basic Figures			
Point: A point has size; it is shown by a and named by a capital			
Line: A line extends on both sides with thickness			
or width; a line is shown with an arrow at ends.			
Segment: A part of a line with two points called; a segment			
shows the two points with arrow at either end.			
Ray: A ray is a part of a line that extends indefinitely in direction; a ray has			
one			
Plane: A plane is a surface that extends indefinitely in all directions and has			
thickness.			

Practice

Use the figure to the right to complete each statement.

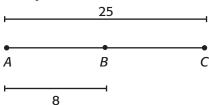
- **1.** The plane shown in the figure is plane _____.
- **2.** The symbol for line *CD* is _____.
- **3.** A ray in the figure can be written using the symbol
- **4.** \overline{PN} is a symbol for ______ PN.



Measuring Segments

Unlike a line, a segment has a beginning point and an ending point, known as **endpoints**. You can measure the distance between the endpoints to find the measure of the segment.

Complete each statement using the figure below.



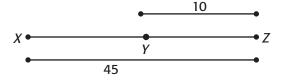
- **1.** The measure of $\overline{AB} = \underline{\hspace{1cm}}$
- **2.** The measure of ____ = 25.
- **3.** The measure of \overline{BC} is \overline{AC} _____ or 25 ____ = ___.
- **4.** So, $\overline{AB} + \overline{BC} = \underline{\hspace{1cm}}$

Complete the rule for segment addition.

If _____ is between *A* and *C*, then
$$\overline{AB} + \overline{BC} = \overline{AC}$$
.
If $\overline{AB} + \overline{BC} = \overline{AC}$, then _____ is between *A* and *C*.

Practice

1. Find XY if Y is between X and Z, if YZ = 10 and XZ = 45.



Write an equation using what you know about segment addition.

$$XY + \underline{\hspace{1cm}} = XZ$$

Plug what you know into the equation.

$$XY + \underline{\hspace{1cm}} = XZ$$

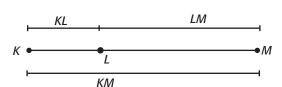
XY + ____ = 45

Solve for the unknown segment length.

Given that L is between K and M, find the missing measure.

3.
$$KL =$$
_____, $LM = 32$, $KM = 47$.

4.
$$KL = 21$$
, $LM =$ ____, $KM = 68$



Name	Date	

Tusing Formulas

In geometry you will use many formulas. There are formulas for finding the area, the volume or perimeter of a figure. A formula is a statement of a relationship between two or more quantities.

Rules for Using Formulas

- **1.** Identify the formula to use. Determine what each variable stands for.
- **2.** Match what you know and don't know from the problem to the variables in the formula.
- **3.** Plug the numbers you know into the formula.
- **4.** If necessary, use order of operations in reverse to undo operations and solve for the unknown variables.

Example

The formula for the area of a triangle is $A = \frac{1}{2}bh$. A triangle has an area of 36 cm² and a height of 12 cm. What is the length of the base?

Step 1 Identify the formula to use. Determine Use the formula given, $A = \frac{1}{2}bh$. what each variable stands for.

A = area, b = base length, h = height

Step 2 Match what you know and don't know You know the area and the height. You need to from the problem to the variables in the find the base length. formula.

$$A = 36 \text{ cm}^2$$
, $b = ?$, $h = 12 \text{ cm}$

Step 3 Plug the numbers you know into the formula.

$$36 = \frac{1}{2} (b)(12)$$

$$6 = b$$

Practice

1. The formula for a trapezoid is $A = \frac{1}{2}h(b_1 + b_2)$. What is the area of a trapezoid with base lengths of 6 and 8 and a height of 10?

Identify the formula to use. Determine what each variable stands for.

The formula given is $A = \frac{1}{2}h(b_1 + b_2)$

Match what you know and don't know from the problem to the variables in the formula.

$$A =$$
 ______, $b_1 =$ one base length $b_2 =$ ______, $h =$ _____

$$A =$$
_____, $b_1 =$ _____, $b_2 =$ _____, $h =$ _____

Plug the numbers you know into the formula.

$$A = \frac{1}{2}$$
 _____ (____ + ____)

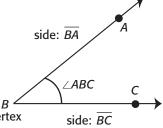
Solve.

- = _____ square units
- **2.** The formula for the volume of a sphere is $V = \frac{4}{3} \pi r^3$. A sphere has a volume of 904 cubic units. What is the radius of the sphere?
- **3.** The area of a parallelogram is 120 in.². The base measurement is 6 inches. What is the length of the height? Use the formula A = bh.

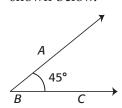
Types of Angles

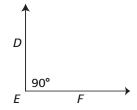
An angle is made of two rays that have a common endpoint. Each ray forms a side of the angle. The common endpoint forms the vertex of the angle.

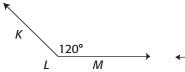
Angles are measured in degrees. An angle's measure is written as $m \angle B = 60^{\circ}$ or $m \angle ABC = 60^{\circ}$

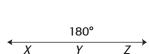


Angles are classified by their measures. Four types of angles are vertex shown below.









Complete the chart below.

Angle Type	Example	Measure
Acute	∠ABC	
Right		90°
	∠KLM	120°
Straight		180°

Complete the statements for the rules for classifying angles.

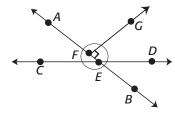
Rules for Classifying Angles

- **1.** An acute angle is an angle whose measure is less than _____.
- **2.** A ______ is an angle whose measure is equal to 90°.
- **3.** An obtuse angle is an angle whose measure is ______90°
- **4.** A ______ is an angle whose measure is equal to 180°.

Practice

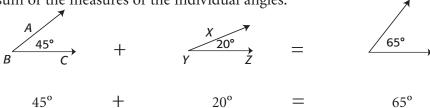
Refer to the figure to answer the following items.

- **1.** $\angle AFG$ has a measure of 90°; $\angle AFG$ is a
 - _____ angle.
- **2.** $\angle AED$ appears to have a measure greater than 90°; $\angle AED$ is an ______ angle.
- **3.** _____ measures 180° and is a straight angle.
- **4.** $\angle DEB$ measures _______ 90°; $\angle DEB$ is an acute angle.

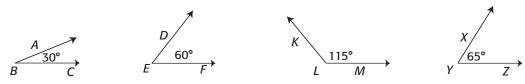


Complementary and Supplementary Angles

Like other measures, you can add angle measures. The result is an angle whose measure is the sum of the measures of the individual angles.



The figures below are two set of angles. Complete the chart below.



Туре	Angle Pair	Measure of One Angle	Measure of the Other Angle	Sum of the Measure
Complementary	∠ABC and ∠DEF	+	=	
Supplementary	∠ <i>KLM</i> and ∠ <i>XYZ</i>	+	=	

Complete the statement for the rules for complementary and supplementary angles.

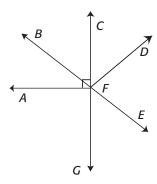
Rules for Complementary and Supplementary Angles

- 1. Two angles are ______ angles if the sum of their measures equals 90°.
- **2.** Two angles are supplementary angles if the sum of their measures equals _____

Practice

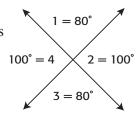
Use the figure to the right to answer the items below.

- **1.** $\angle AFB$ is complementary to _____.
- **2.** $m \angle CFE + m \angle EFG = \underline{\hspace{1cm}}$
- **3.** $\angle BFC = 35^\circ$; $\angle BFC$ and $\angle CFE$ are supplementary. What is the measure of $\angle CFE$?
- **4.** ∠*BFC* and _____ are complementary angles.



Pairs of Angles

As you know when two lines intersect four angles are created, as you can see in the figure on the right. Certain relationships exist among the angles formed by intersecting lines.



Complete the chart below.

Туре	Measure of One Angle	Measure of the Other Angle
Vertical Angles	m∠1 =	m∠3 =
	m∠2 =	m∠4 =
Linear Pair	m∠1 =	m∠2 =
	m∠3 =	m∠4 =

Complete the statements below.

- **1.** $\angle 1$ and $\angle 3$ are ______, ____ and _____ are also vertical angles.
- **2.** $\angle 1$ and $\angle 2$ form a ______, ____ and _____ also form a linear pair.
- **3.** The sum of the measures of $\angle 1$ and $\angle 2$ is ______; the sum of the measures of _____ and ____ is 180° .
- **4.** Another term for a linear pair is ______ angles.

Complete the statements for rules for angle pairs.

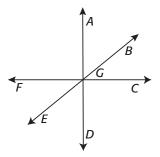
Rules for angle pairs

- **1.** When two lines intersect, ______ angles are created opposite one another.
- **2.** Vertical angles have ______ measure; they are ______.
- **3.** The sum of the measures of the angles in a linear pair is _____.

Practice

Use the figure to the right to complete the following statements.

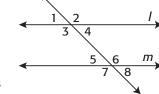
- **1.** $\angle AGB$ and _____ are vertical angles.
- **2.** $\angle AGB$ and $\angle BGD$ are ______.
- **3.** The measure of $\angle FGE$ is 45°. The measure of $\angle EGC$ is _____.
- **4.** $\angle EGD$ is supplementary to \angle _____.
- **5.** An angle congruent to $\angle DGC$ is _____.



Parallel Lines: Types of Angles

In the figure to the right, lines l and m are parallel lines. Line t intersects lines l and m, line t is known as a transversal.

A **transversal** is a line that intersects two or more lines at different points.



As you can see, when a transversal intersects two lines, eight angles are formed. These angles are given special names.

Complete the rules below by using the diagram above.

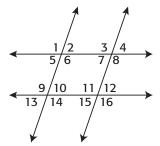
Rules for Angles Formed by Parallel Lines Intersected by a Transversal

- **1.** Exterior angles are angles on the outside of the lines; angles 1, 2, _____, and _____ are exterior angles.
- **2.** Interior angles are angles on the inside of the lines; angles 3, 4, _____, and _____ are interior angles.
- **3.** Consecutive interior angles are angles that are inside the lines on the same side of the transversal; angles 3 and 5 and angles _____ and ____ are consecutive interior angles.
- **4.** Alternate interior angles are angles that are inside the lines but on the opposite sides of the transversal; angles 3 and 6 and angles _____ and ____ are alternate interior angles.
- **5.** Alternate exterior angles are angles that are outside the lines on opposite sides of the transversal; angles 1 and 8 and angles _____ and ____ are alternate exterior angles.
- **6.** Corresponding angles occupy the same position on each line; angles 1 and 5 and angles 3 and 7 are corresponding angles, as are angles 2 and _____ and angles ____ and 8.

Practice

Classify each pair of angles using the figure to the right.

- **1.** ∠7 and ∠12 _____
- **2.** ∠1 and ∠13 _____
- **3.** ∠11 and ∠14 _____
- **4.** ∠4 and ∠5 _____



Identify the missing angle in each pair.

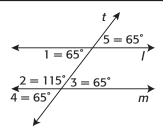
- **5.** Corresponding angles: ∠3 and ∠____ or ∠____
- **6.** Consecutive interior angles: ∠6 and ∠____ or ∠____
- **7.** Interior angles: ∠10 and ∠____ or ∠____

Parallel Lines: Angle Relationships

In the figure to the right lines l and m are parallel lines. Line t intersects line l and m, line t is known as a transversal.

A **transversal** is a line that intersects two or more lines at different points.

As you can see, when a transversal intersects two lines, 8 angles are formed. These angles have special relationships.



Explore the angle relationships that exist when a transversal intersects two parallel lines.

Туре	Measure of Angle	Measure of Other Angle
Corresponding angle	m∠1 = 65°	
Alternate interior angles		m∠3 = 65°
	m∠1 = 65°	m∠2 = 115°
Alternate exterior angles	m∠5 = 65°	

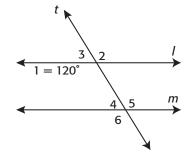
Use the chart to complete the statements below.

Rules for the relationships among angles formed when a transversal intersects parallel lines

- **1.** Corresponding angles are ______.
- 2. Alternate interior angles are _____
- **3.** _____ angles are supplementary.
- **4.** Alternate exterior angles are ______.

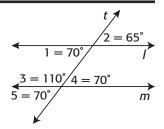
Practice

Use the figure to the right.



Proving Lines Are Parallel

In the figure to the right, lines *l* and *m* are parallel lines. When a transversal intersects two lines, 8 angles are formed. These angles have special relationships. You can use these relationships to prove lines are parallel.



Use the figure above to help complete the following statements.

Rules for Proving Lines are Parallel

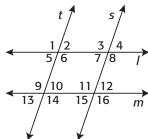
- **1.** If two lines are intersected by a transversal and ______ angles, such as $\angle 1$ and $\angle 5$, are ______, then the lines are parallel.
- **2.** If two lines are intersected by a transversal and ______ angles, such as $\angle 1$ and $\angle 4$, are ______, then the lines are parallel.
- **3.** If two lines are intersected by a transversal and ______ angles, such as $\angle 2$ and $\angle 5$, are ______, then the lines are parallel.
- **4.** If two lines are intersected by a transversal and ______ angles, such as $\angle 1$ and $\angle 3$, are ______, then the lines are parallel.

Example

State the rule that says why the lines are parallel, $m \angle 5 \cong \angle 10$

- **Step 1** State the relationship between $\angle 5$ and $\angle 10$. The angles are alternate interior angles.
- **Step 2** State how the measures of each angle are related. The angles are congruent.
- **Step 3** State why the lines *l* and *m* are parallel.

 If two lines are intersected by a transversal and alternate interior angles are congruent, then the lines are parallel.



Practice

State the rule that says why the lines are parallel. Use the figure above.

1. $m \angle 5 + m \angle 9 = 180^{\circ}$

State the relationship between ∠5 and ∠9: The angles are ______ angles.

State how the measure of each angle is related: The angles are ______.

State why lines *l* and *m* are parallel: If two lines are intersected by a transversal and _______, the lines are parallel.

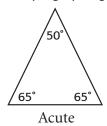
- **2.** m∠4 ≅ m∠5 _____
- **3.** m∠5 ≅ m∠7 _____
- **4.** m∠8 ≅ m∠11 _____
- **5.** $m \angle 10 + m \angle 11 = 180^{\circ}$

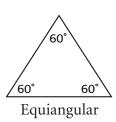
Classifying Triangles

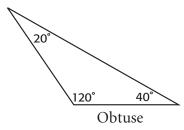
A triangle is a three-sided polygon. A polygon is a closed figure made up of segments, called sides, that intersect at the end points, called vertices.

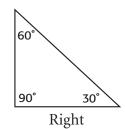
Triangles are classified by their angles and their sides.

Classifying by angle:

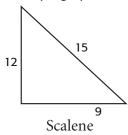


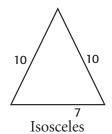


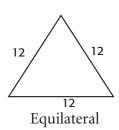




Classifying by side length:







Use the figures above to complete the rules for classifying triangles.

Rules for classifying triangles by angle

- 1. An acute triangle has ______ acute angles.
- **2.** An equiangular triangle has three _____ angles.
- **3.** An obtuse triangle has one _____ angle.
- **4.** A right triangle has one _____ angle.

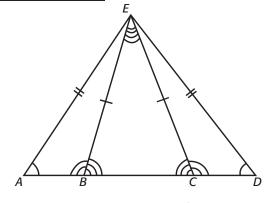
Rules for classifying triangles by side length

- **1.** A scalene triangle has _____ congruent sides.
- **2.** An isosceles triangle has at least _____ congruent sides.
- **3.** An equilateral triangle has _____ congruent sides.

Practice

Use the figure to the right.

- 1. Name an equilateral triangle.
- **2.** Name a scalene triangle.
- **3.** Name an obtuse triangle.
- **4.** Name an acute triangle.
- **5.** Name an isosceles triangle.



Interior and Exterior Angles in Triangles

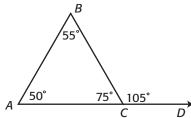
A triangle is a three-sided polygon. A triangle is made of segments, called sides, that intersect only at their endpoints, called vertices.

Sides: \overline{AB} , \overline{BC} , \overline{CA}

Vertices: A, B, C

Interior Angles: $\angle BAC$, $\angle ABC$, $\angle BCA$

Exterior Angle: $\angle BCD$



Complete each statement below.

- **1.** The measure of $\angle BAC$ is _____, the measure of $\angle ABC$ is _____, and the measure of $\angle BCA$ is _____. If you add the measures of the interior angles, the sum is _____.
- **2.** $\angle BCD$ is an exterior angle. The measure of $\angle BCD$ is ______. $\angle BAC$ and $\angle ABC$ are both known as remote interior angles. The measure of $\angle BAC$ is _____ and the measure of $\angle ABC$ is ______. If you add the measures of these remote interior angles, the sum is ______.
- **3.** The measure of $\angle BCA$ is _____. The measure of $\angle BCD$ is _____. If you add the measure of $\angle BCA$ and $\angle BCD$, the sum is _____; the angles are _____.

Use the figure above to complete the rules for angle relationships in triangles.

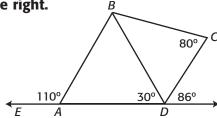
Rules for Angle Relationships in Triangles

- **1.** The sum of the measures of the interior angles of a triangle is _____. $m\angle 1 + m\angle 2 + m\angle 3 =$ _____
- 2. The measure of an ______ angle of a triangle is _____ to the sum of the measures of the two remote interior angles.

Practice

Find the measures of the angles in the figure to the right.

- **1.** m ∠*ABD* = _____
- **2.** m ∠*BAD* = _____
- **3.** m ∠*CDB* = _____
- **4.** m ∠*CBD* = _____



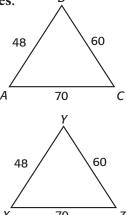
Corresponding Parts of Triangles

Triangles that are the same size and the same shape are **congruent triangles**.

As you know each triangle has six parts—three sides and three angles.

Use the figures to the right to identify corresponding parts. Use the symbol "←→" to mean "corresponds to".

$$\angle CAB \longleftrightarrow \angle ZXY$$
 $\overline{AC} \longleftrightarrow \overline{XZ}$
 $\angle ABC \longleftrightarrow \underline{\qquad} \overline{AB} \longleftrightarrow \underline{\qquad} \overline{YZ}$



Complete the chart below.

Angle	Corresponding Angle	Relationship
∠ <i>CAB</i> = 70°	∠ <i>ZXY</i> = 70°	$\angle CAB \cong \angle ZXY$
∠ <i>ABC</i> = 57°		∠ABC ≅
	∠ <i>YZX</i> = 53°	≅ ∠YZX

Side	Corresponding Side	Relationship
ĀC	XZ	$\overline{AC} \cong \overline{XZ}$
ĀB		<i>ĀB</i> ≅
BC	<u>YZ</u>	≅ <u>YZ</u>

Complete the statement below for the rules for corresponding parts of congruent triangles.

Rules for Corresponding Parts of Corresponding Triangles

Two triangles are congruent if and only if their ______ parts

Practice

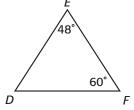
Complete each statement. $\triangle DEF \cong \triangle RST$

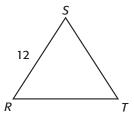
1.
$$\overline{DE} = \underline{\hspace{1cm}}$$

3.
$$m \angle RST =$$

4.
$$\overline{ST} \cong \underline{\hspace{1cm}}$$

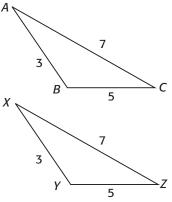
6.
$$\overline{DF} =$$





Triangle Congruence: Side-Side Congruence

If two triangles have three pairs of congruent corresponding angles and three pairs of congruent corresponding sides, then the triangles are congruent. However, you do not need to know that all the sides and angles are congruent in order to conclude that the triangles are congruent. You only need to know certain corresponding parts are congruent. The two triangles to the right are congruent.



Complete the chart by identifying the corresponding sides and their measures.

Side	Measure	Corresponding Side	Measure	Relationship Between Sides
ĀB	3	XY	3	$\overline{AB} \cong \overline{XY}$
BC	5			<u>BC</u> ≅
		XZ	7	≅ XZ

Complete the rule for triangle congruence.

Rule for Side-Side (SSS) Postulate

If three sides of one triangle are ______ to _____

sides of another triangle then the two triangles are congruent.

Practice

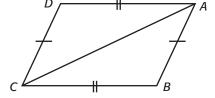
For each figure, determine if there is enough information to prove the two triangles congruent.

1. The corresponding side to side *AB*: _____

Are the sides congruent? _____

The corresponding side to side *AD*: _____

Are the sides congruent? _____

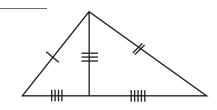


What do you notice about side AC? _____

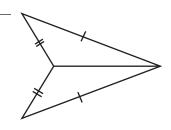
Does \overline{AC} in $\triangle ABC$ correspond to \overline{AC} in $\triangle ACD$?

Can you use SSS postulate? _____

2.



3.

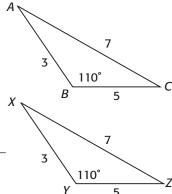


Triangle Congruence: Side-Angle-Side Congruence

You do not need to know that all the sides and angles are congruent in order to conclude that triangles are congruent. You only need to know certain corresponding parts are congruent.

The two triangles to the right are congruent. Use them to answer the following.

- **1.** Which sides in triangle ABC form $\angle B$? \overline{AB} and _____
- **2.** Which angle is formed from (included) \overline{AB} and \overline{AC} ?
- **3.** Which two angles are made using side AB?



The two triangles above are congruent. Complete the chart by identifying the corresponding sides and angles and their measures.

Side	Measure	Corresponding Side	Measure	Relationship Between Sides
ĀB	3		3	$\overline{AB} \cong \overline{XY}$
Angle	Measure	Corresponding Angle	Measure	Relationship Between Angles
	110°	∠ Y	110°	≅ ∠Υ
Side	Measure	Corresponding Side	Measure	Relationship Between Sides
	5			<u>BC</u> ≅

Complete the rule for triangle congruence.

Rule for Side-Angle-Side (SAS) Postulate.

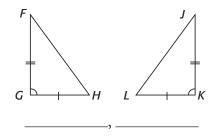
If the two sides and the included angle of one triangle are _______ to two _____ and the ______ angles, then the triangles are congruent.

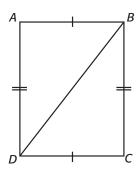
Practice

Name the included angle between each pair of sides.

- **1.** \overline{AD} and \overline{AB} _____
- **2.** \overline{BD} and \overline{BC}
- **3.** \overline{BC} and \overline{DC}

4.



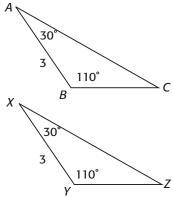


Triangle Congruence: Angle-Side-Angle Congruence

You do not need to know that all the sides and angles are congruent in order to conclude that triangles are congruent. You only need to know certain corresponding parts are congruent.

The two triangles to the right are congruent. Use them to answer the following questions.

- **1.** Which side is included by $\angle A$ and $\angle C$?
- **2.** Which side is included by $\angle B$ and $\angle C$?
- **3.** Which side is included by $\angle A$ and $\angle B$?



The two triangles above are congruent. Complete the chart by identifying corresponding sides and angles and their measures.

Angle	Measure	Corresponding Angle	Measure	Relationship Between Angles
∠B	110°	∠ <i>Y</i>	110°	∠ <i>B</i> ≅ ∠ <i>Y</i>
Side	Measure	Corresponding Side	Measure	Relationship Between Sides
AB	3			<i>ĀB</i> ≅
Angle	Measure	Corresponding Angle	Measure	Relationship Between Angles
		∠X	30°	≅ ∠X

Complete the rule for triangle congruence.

Rule for Angle-Side-Angle (ASA) Postulate.				
If the two angles and the included side of one triangle are				
to two and the	side of another triangle, then			
the two triangles are congruent.				

Practice

State the missing congruence that must be given to use the ASA Postulate to prove the triangles are congruent.

1. Which pair of corresponding angles are given? __ Which set of corresponding sides are given? _ Which angles are adjacent to \overline{AC} ? _____

If $\triangle ABC$ and $\triangle XYZ$ are congruent by ASA, which is the other angle in $\triangle ABC$?

